1. A crime is committed by one of two suspects, A and B. Initially, there is equal evidence against both of them. In further investigation at the crime scene, it is found that the guilty party had a blood type found in 10% of the population. Suspect A does match this blood type, whereas the blood type of Suspect B is unknown. (a) Given this new information, what is the probability that A is the guilty party? (b) Given this new information, what is the probability that B’s blood type matches that found at the crime scene?

Answer:

Let's define the following events:

* G: the event that A is guilty
* B: the event that the blood type of the guilty person matches the blood type found at the crime scene
* M: the event that A matches the blood type found at the crime scene

From the problem statement, we know that P(G) = P(Gc) = 0.5, P(B|G) = 0.1, and P(M|G) = 1. We want to find:

(a) P(G|M): the probability that A is guilty given that their blood type matches that found at the crime scene.

By Bayes' Theorem, we have:

P(G|M) = P(M|G)P(G) / P(M)

We can find the denominator using the Law of Total Probability:

P(M) = P(M|G)P(G) + P(M|Gc)P(Gc) = 1 x 0.5 + 0.1 x 0.5 = 0.55

Substituting in the values, we get:

P(G|M) = (1 x 0.5) / 0.55 = 0.909

Therefore, given that A matches the blood type found at the crime scene, the probability that A is guilty is 0.909.

(b) P(B|Bc): the probability that B's blood type matches the blood type found at the crime scene.

By the Law of Total Probability, we have:

P(B) = P(B|G)P(G) + P(B|Gc)P(Gc) = 0.1 x 0.5 + 0 x 0.5 = 0.05

We want to find P(B|Bc), the probability that B's blood type matches the blood type found at the crime scene given that A's blood type matches the blood type found at the crime scene. We can use Bayes' Theorem:

P(B|Bc) = P(Bc|B)P(B) / P(Bc)

The probability that the blood type does not match the one found at the crime scene, P(Bc), is given by:

P(Bc) = P(Bc|G)P(G) + P(Bc|Gc)P(Gc) = 0.9 x 0.5 + 1 x 0.5 = 0.95

Substituting in the values, we get:

P(B|Bc) = (P(Bc|B)P(B)) / P(Bc) = (0.1 x 0.05) / 0.95 = 0.00526

Therefore, given that A's blood type matches the blood type found at the crime scene, the probability that B's blood type matches the blood type found at the crime scene is 0.00526.